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ABSTRACT

Commonality analysis is a method of partitioning variance that has advantages over more traditional "OVA" methods. Commonality analysis indicates the amount of explanatory power that is "unique" to a given predictor variable and the amount of explanatory power that is "common" to or shared with at least one predictor variable. This paper outlines and discusses the steps of commonality analysis specific to canonical correlation analysis using a heuristic example to make the discussion more concrete. The first step in canonical commonality analysis is to perform a canonical correlation analysis in order to derive the standardized canonical function coefficients and the canonical functions. Step two is to calculate the criterion composite scores, also called variate scores. The third step is to conduct multiple regression on the synthetic composite criterion variables using all possible combinations of the predictor variables. The final step is to calculate the unique and common variance partitions. The advantages and potential limitations of commonality analysis are discussed. An appendix presents a computer program for the analysis. (Contains four tables and seven references.) (Author/SLD)

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# Canonical Commonality Analysis

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Abstract

Commonality analysis is a method of partitioning variance that has advantages over more traditional "OVA" methods. Specifically, commonality analysis indicates the amount of explanatory power that is "unique" to a given predictor variable and the amount of explanatory power that is "common" to or shared with at least one other predictor variable. This paper outlines and discusses the steps of commonality analysis specific to canonical correlation analysis using a heuristic example to make the discussion more concrete. Additionally, advantages of commonality analysis are provided, as well as its potential limitations.

## Canonical Commonality Analysis

Recent reviews of educational research indicate that "OVA" techniques are being used less frequently as the statistical method of choice, although they are still quite popular (Goodwin & Goodwin, 1985; Willson, 1980). For example, Elmore and Woehlke (1988) reported that ANOVA and ANCOVA were used in about 25% of the research articles published in three educational journals from 1978 to 1987. One potential reason why such "OVA" techniques maintain their popularity is that they allow the researcher to divide the dependent variable into a number of portions, including the main effects of each independent variable and the interactions between the independent variables (Daniel, 1989). However, 'OVA" methods are not the only processes by which the researcher can partition the variance of the criterion variable. Commonality analysis offers another method of doing so with certain advantages over more traditional methods.

What is commonality analysis? Commonality analysis indicates the amount of explanatory power that is "unique" to a given predictor variable and the amount of explanatory power that is "common" to or shared with at least one other predictor variable (Thompson & Miller, 1985). This analysis may be used in conjunction with the canonical correlation method to aid in the interpretation of canonical results.

Commonality analysis may be conducted in both univariate and multivariate analyses. The only difference between univariate and multivariate commonality analyses is that in the multivariate case, the criterion variables must be converted to synthetic composite scores (Daniel, 1989), which will be discussed in more detail below. The purpose of this paper is to address commonality analysis specific to canonical correlation analysis. A heuristic example is provided to make the discussion more concrete.

### Steps of Canonical Commonality Analysis

The first step in canonical commonality analysis is to perform a canonical correlation analysis in order to derive the standardized canonical function coefficients and the

canonical functions. In this example, the analysis included two criterion variables ("catmed" and "moress") and two predictor variables ("lessfed" and "moredef").

Insert Table 1 about here.

Step two in this analysis is to calculate the criterion composite scores, also called "variate" scores. To do so, the standardized canonical function coefficients are multiplied by the Z-scores on the criterion variables. These products are then summed to create the synthetic criterion composite variables - one for each function yielded by the canonical correlation analysis. For example, the computations for the two functions would be:

crit1 = (-0.752 x zcatmed) + (1.572 x zmoress)

crit2 = (1.820 x zcatmed) + (-1.187 x zmoress)

The third step is to conduct a multiple regression on the synthetic composite criterion variables using all possible combinations of the predictor variables. It should be noted that the squared correlation coefficient when all the predictors are used simultaneously (regression result) always equals the squared canonical correlation because the two analyses are the same in the full model case (Daniel, 1989; Thompson & Miller, 1985).

The final step in the commonality analysis is to calculate the unique and common variance partitions. Because there are two predictor variables, there are three possible unique and common commonality components (Rowell, 1991). Table 2 lists formulas for calculating unique and commonality components with different numbers of predictor variables.

Insert Table 2 about here.

Table 3 shows calculations for the unique and commonality components for both functions in this example.

Insert Table 3 about here.

Furthermore, the explanatory power of each predictor is calculated by adding down the columns for each predictor. Table 4 displays the commonality results for each function.

Insert Table 4 about here.

In this example, it is shown that on the first function, the majority of the explanatory power of predictors is common to both variables. Although 64.0% of the variance can be accounted for by the variable "Moredef" alone on the first function, 48.3% of the total explanatory power is common to both predictors. The unique explanatory power of "Moredef" is therefore only 15.7%. The unique explanatory power of "Lessfed" is only 7.0% (.553 - .48313). Also, it can be seen that the second function has virtually no explanatory ability.

## Discussion

A couple of points need to be addressed regarding commonality analysis. First, as may be obvious from the formulas provided in Table 2, the number of components or variance partitions increases rapidly as additional predictors are considered (Rowell, 1991). Not only does this make the calculations more tedious, but severely complicates interpretation of the results as well. Many have recommended that only four predictors be used or that some predictors may be grouped together into meaningful subsets to reduce the number of predictors.

Second, negative commonalities indicate the presence of suppressor effects (Thompson & Miller, 1985). Beaton (1973) explained that the negative commonality, therefore, indicates that the explanatory power of either predictor variable is greater when the other is used.

Finally, it is important to understand the limitations and advantages of commonality analysis, as with any method. The main limitation often cited is that there is no statistical significance test for commonalities. However, this fact is not necessarily detrimental

because commonality analysis is generally conducted after a significant canonical correlation has already been found. A second potential limitation may be the fact that this procedure can only realistically accommodate a limited number of predictor variables for the reasons previously addressed. Although this technique is superior to others that do not examine the partitioning of explained variance, it is still somewhat limited in its ability to do so.

Despite these limitations, commonality analysis has several advantages, as cited by Daniel (1989) and Thompson and Miller (1985). First, commonality analysis honors that relationships among variables because the variables do not need to be converted from their original scale and because it analyzes all possible orders of entry of the predictors. Second, because commonality indicates the degree of overlap of variables, it may be helpful in the social sciences where variables are often correlated with one another. Third, because commonality analysis was originally a regression technique that was extended to the canonical case, it reinforces the idea that canonical correlation analysis is the most general linear model of parametric statistics.

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Table 1

Original Data

| CATMED | MORESS | LESSFED | MOREDEF |
|---|---|---|---|
| 15 | 16 | 20 | 20 |
| 14 | 14 | 19 | 19 |
| 12 | 13 | 10 | 11 |
| 14 | 13 | 9 | 10 |
| 15 | 15 | 8 | 9 |
| 15 | 14 | 7 | 8 |
| 17 | 16 | 20 | 19 |
| 13 | 15 | 19 | 19 |
| 15 | 16 | 18 | 19 |
| 14 | 16 | 17 | 17 |
| 10 | 12 | 15 | 15 |
| 10 | 11 | 8 | 8 |
| 10 | 9 | 8 | 6 |
| 14 | 15 | 18 | 17 |
| 13 | 13 | 10 | 10 |
| 15 | 15 | 17 | 17 |
| 16 | 16 | 20 | 19 |
| 14 | 15 | 19 | 20 |
| 14 | 14 | 16 | 16 |
| 13 | 13 | 10 | 9 |
| 11 | 12 | 15 | 15 |
| 13 | 12 | 9 | 9 |

Table 2

Formulas for Unique and Commonality Components of Variance

Two Predictor Variables

$$U1 = -R^2(2) + R^2(12)$$
$$U2 = -R^2(1) + R^2(12)$$
$$C12 = R^2(1) + R^2(2) - R^2(12)$$

Three Predictor Variables

$$U1 = -R^2(23) + R^2(123)$$
$$U2 = -R^2(13) + R^2(123)$$
$$U3 = -R^2(12) + R^2(123)$$
$$C12 = -R^2(3) + R^2(13) + R^2(23) - R^2(123)$$
$$C13 = -R^2(2) + R^2(12) + R^2(23) - R^2(123)$$
$$C23 = -R^2(1) + R^2(12) + R^2(13) - R^2(123)$$
$$C123 = R^2(1) + R^2(2) + R^2(3) - R^2(12) - R^2(13) - R^2(23) + R^2(123)$$

Four Predictor Variables

$$U1 = -R^2(234) + R^2(1234)$$
$$U2 = -R^2(134) + R^2(1234)$$
$$U3 = -R^2(124) + R^2(1234)$$
$$U4 = -R^2(123) + R^2(1234)$$
$$C12 = -R^2(34) + R^2(134) + R^2(234) - R^2(1234)$$
$$C13 = -R^2(24) + R^2(124) + R^2(234) - R^2(1234)$$
$$C14 = -R^2(23) + R^2(123) + R^2(234) - R^2(1234)$$
$$C23 = -R^2(14) + R^2(124) + R^2(134) - R^2(1234)$$
$$C24 = -R^2(13) + R^2(123) + R^2(134) - R^2(1234)$$
$$C34 = -R^2(12) + R^2(123) + R^2(124) - R^2(1234)$$
$$C123 = -R^2(4) + R^2(14) + R^2(24) + R^2(34) - R^2(124) - R^2(134) - R^2(234) + R^2(1234)$$
$$C124 = -R^2(3) + R^2(13) + R^2(23) + R^2(34) - R^2(123) - R^2(134) - R^2(234) + R^2(1234)$$
$$C134 = -R^2(2) + R^2(12) + R^2(23) + R^2(24) - R^2(123) - R^2(124) - R^2(234) + R^2(1234)$$
$$C234 = -R^2(1) + R^2(12) + R^2(13) + R^2(14) - R^2(123) - R^2(124) - R^2(134) + R^2(1234)$$
$$C1234 = R^2(1) + R^2(2) + R^2(3) + R^2(4) - R^2(12) - R^2(13) - R^2(14) - R^2(23) - R^2(24) - R^2(34) + R^2(123) + R^2(124) + R^2(134) + R^2(234) - R^2(1234)$$

Note. From "Partitioning predicted variance components into constituent parts: How to conduct commonality analysis," by K. Rowell, 1991.

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Table 3

Example Calculations of Variance Partitions

Function 1

$U1 = -R^2(2) + R^2(12) = -.64018 + .71057 = .15705$

$U2 = -R^2(1) + R^2(12) = -.55352 + .71057 = .07039$

$C12 = R^2(1) + R^2(2) - R^2(12) = .55352 + .64018 - .71057 = .48313$

Function 2

$U1 = -R^2(2) + R^2(12) = -.00003 + .00034 = .00031$

$U2 = -R^2(1) + R^2(12) = -.00007 + .00034 = .00027$

$C12 = R^2(1) + R^2(2) - R^2(12) = .00007 + .00003 - .00034 = -.00024$

Table 4

Commonality Analysis Summary Tables - for each function

Function 1

| Component | Lessfed | Moredef |
|---|---|---|
| Unique to Lessfed | .07039 | |
| Unique to Moredef | | .15705 |
| Common to Lessfed/ Moredef | .48313 | .48313 |
| Sum of Components | 55.352% | 64.018% |
| $r^2$ of prediction with canonical composite scores | .55352 | .64018 |

Function 2

| Component | Lessfed | Moredef |
|---|---|---|
| Unique to Lessfed | .00031 | |
| Unique to Moredef | | .00027 |
| Common to Lessfed/ Moredef | -.00024 | -.00024 |
| Sum of Components | .007% | .003% |
| $r^2$ of prediction with canonical composite scores | .00007 | .00003 |

Appendix

```
LIST
  VARIABLES=CATMED MORESS LESSFED MOREDEF
  /CASES=BY 1
  /FORMAT=WRAP UNNUMBERED.
MANOVA
  CATMED MORESS WITH LESSFED MOREDEF
  /PRINT=SIGNIF (MUT EIGEN DIMENR)
  /DISCRIM=STAN CORR ALPHA (.99)
  /DESIGN.
DESCRIPTIVES VARIABLES=ALL/SAVE.
LIST VARIABLES=ALL/CASES=22/FORMAT=NUMBERED.
COMPUTE CRIT1=(-0.752*ZCATMED) + (1.572*ZMORESS).
COMPUTE CRIT2=(1.820*ZCATMED) + (-1.187*ZMORESS).
DESCRIPTIVES VARIABLES=ALL.
SUBTITLE '1a REGRESSION TO PRED CANONICAL SYN WITH 2 PREDS'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT1/
  ENTER LESSFED MOREDEF.
SUBTITLE '1b REGRESSION TO PRED CANONICAL SYN WITH 2 PREDS'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT2/
  ENTER LESSFED MOREDEF.
SUBTITLE '2a REGRESSION TO PRED CANONICAL SYN WITH LESSFED'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT1/
  ENTER LESSFED.
SUBTITLE '2b REGRESSION TO PRED CANONICAL SYN WITH LESSFED'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT2/
  ENTER LESSFED.
SUBTITLE '3a REGRESSION TO PRED CANONICAL SYN WITH MOREDEF'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT1/
  ENTER MOREDEF.
SUBTITLE '3B REGRESSION TO PRED CANONICAL SYN WITH MOREDEF'.
REGRESSION VARIABLES=CRIT1 CRIT2 LESSFED MOREDEF/DEPENDENT=CRIT2/
  ENTER MOREDEF.
```